# MATH 147 QUIZ 9 SOLUTIONS

1. Find the length of one turn of the helix given by $\mathbf{r}(i) = \frac{1}{2}\cos(t)\vec{i} + \frac{1}{2}\sin(t)\vec{j} + \sqrt{\frac{3}{4}}t\vec{k}$. (5 Points)

   To find the length of a curve, we integrate $\int_C \|\mathbf{r}'(t)\} \, dt$. Thus, we first find $\mathbf{r}'(t)$, which is

   $$\mathbf{r}'(t) = (-1/2\sin(t), 1/2\cos(t), \sqrt{3/4}).$$

   Next, we find the magnitude of this, which will be

   $$\|\mathbf{r}'(t)\| = \sqrt{\frac{1}{4}\sin^2(t) + \frac{1}{4}\cos^2(t) + \frac{3}{4}} = \sqrt{1} = 1.$$

   Thus, as one turn is $0 \leq t \leq 2\pi$, we should get that the arc length is $\int_0^{2\pi} \, dt = 2\pi$.

2. Calculate $\int_C \frac{x+y}{y+z} \, dt$ for C given by $\mathbf{r}(t) = t\vec{i} + \frac{2}{3}t^{3/2}\vec{j} + t\vec{k}$ with $1 \leq t \leq 2$. (5 points)

   We again find the magnitude of the tangent vector to the curve. We have $\mathbf{r}'(t) = (1, \sqrt{t}, 1)$, and so $\|\mathbf{r}'(t)\| = \sqrt{t+2}$. Thus, our integral is

   $$\int_C \frac{x+y}{y+z} \, dt = \int_1^2 \frac{t + \frac{2}{3}t^{3/2}}{\frac{2}{3}t^{3/2} + t}\sqrt{t+2} \, dt = \int_1^2 \sqrt{t+2} \, dt = \frac{2}{3}(t+2)^{3/2}\Big|_1^2 = \frac{16}{3} - \frac{2 * 3^{3/2}}{3} = \frac{16}{3} - 2\sqrt{3}.$$